MODELING, PREDICTION AND FORECAST OF THE IONOSPHERIC CRITICAL FREQUENCY foF2

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SUMMARY: *The temporal variations of the ionospheric critical frequency foF2 is a typical example of a time series in which deterministic variations at various time scales and nonstationary stochastic variations are involved. In a series of journal and conference papers we have obtained various modelling, prediction and forecast algorithms for foF2 variations over Europe, based on data from about 15 ionosonde stations over 41 years. We recovered the well known results that variations in the monthly medians obey a linear or parabolic regression model in terms of the smoothed sunspot number* R_{12}. *A model involving a trigonometric expansion in the harmonics of the annual variation modulated by* R_{12} *is constructed. For prediction of the monthly medians, we used this model with a "sliding window", based on 48 months of observation to predict the foF2 for the forthcoming month, with an error of about %3-4. Statistical properties of the deviations from monthly medians were studied and we used a one-parameter feedback to forecast the hourly values of foF2 was used.*

KEYWORDS: *foF2, ionospheric critical frequency, modelling, prediction, forecast, feedback.*

ÖZET: *İyonosferik kritik frekans verisi foF2, çeşitli ölçeklerde determinisitik süreçler ile durağan olmayan stokastik süreçlerin bir arada bulunduğu bir zaman serisi örneğini oluşturmaktadır. Çeşitli dergilerde yayımlanan ve konferanslarda sunulan bir dizi çalışmada, foF2 verisinin Avrupa üzeindeki değişimleri için çeşitli modeller geliştirilmiş, öngörü ve kestirim çalışmaları yapılmıştır. Çalışmalarda 15 istasyonun yaklaşık 41 yıllık verisi kullanılmıştır. Modellemede, yavaş değişimler için düzgünleştirilmiş güneş lekeleleri sayısına bağlı bilinen lineer veya parabolik modellerin yanı sıra yıllık değişimin harmoniklerinin* R_{12} *ile modülasyonu kullanımıştır. Öngörü için bu model esas alınmış, 48 ay uzunluklu bir "kayan pencere" kullanılarak bir sonraki ayın aylık medyan değeri %3-4 hata ile bulunmuştur. Daha sonra saatlik foF2 değerlerinin aylık medyan değerlerden farknın istatistik özellikleri ı incelenmiş ve bir parametreli bir geri-besleme yöntemi ile saatlik foF2 değerlerinin kestirimi yapılmıştır.*

ANAHTAR KELİMELER: *foF2, iyonosfer, kritik frekans, modelleme, öngörü, kestirim.*

1. INTRODUCTION

The ionospheric critical frequency foF2 indicates the largest allowable frequency of waves that can propagate on Earth, by reflection from the ionosphere. Thus the knowledge of spatial and temporal varitions of foF2 is crucial in radio communications. The investigation of foF2 variations is part of the research on ionospheric variability, studied in the framework of various European Union Actions COST 251 and 271. We have used the foF2 data of a number of European ionosonde stations over the period of 1958-1996, from high (56.4N-

67.8N) to mid (45.5N-55.5N), and low (37.9N-42.7N) latitudes, including Kiruna, Lycksele, Arkangelsk, Uppsala, Sverdlovsk, Moscow, Kaliningrad, Kiev, Slough, Dourbes, Lannion, Poitiers, Novokazalinsk, Rome and Ashkhabad, some of which shown in the map below

Figure 1. The geographic coordinates of the stations used in the study.

The foF2 variations in the large follow very closely the variations in the sunspot numbers. Sunspot numbers are observed since late 1600's and are one of the longest series of reliable solar data. They have a period of nearly 11 years and are identified with cycle numbers. It is well known that the 12-month average of these numbers called R_{12}, gives better results compared with the actual monthly or daily variations.

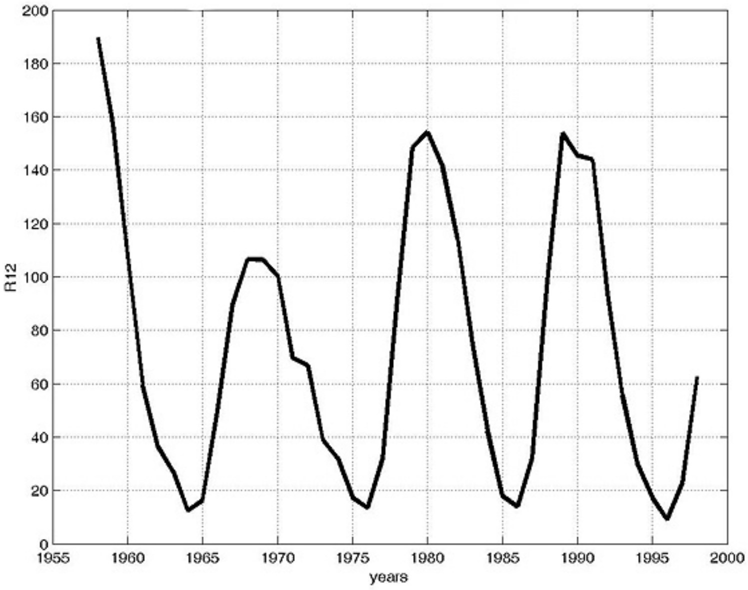


Figure 2. The change of R_{12} for years between 1958-1998

Ionospheric data is available at the World Data Center's database and foF2 data can be downloaded for various stations over all available periods of time. Our work was based on data arranged into monthly files for each station. These files involved information on the quality of the data and missing values was left blank. We used standard MATLAB functions for the analysis of the data. The handling of multiple files and the conversion to MATLAB format was the most time-consuming part of the analysis. Missing data was filled by various averaging techniques and outliers were eliminated before processing.

We present below main results of our work, which is grouped as modelling for a single station, description of the sliding window technique for the prediction of monthly medians and the description of the feedback method for forecast, respectively in section 2, 3, and 4.

This paper is based on the works in reference [1-6]

2. SINGLE STATION MODELING

For this study, we have used monthly mean values of the critical frequency foF2 from 12 ionosonde stations for the years 1970-1989. The average values were more or less complete except for a four year gap in Rome and a two year gap in Ashkabad data which were filled by interpolation. The geographic latitudes and longitudes of each station on the correlation coefficients of foF2 values at these stations with R_{12} are given below

Station	**Latitude**	**Longitude**	**Correlation with R_{12}**
Kiruna	67,8N	20,4E	
Lycksele	64,6N	18,8E	
Arkhangelsk	64,6N	40,5E	0.8981
Uppsala	59,8N	17,6E	0.8772
Sverdlovsk	56,4N	58,6E	0.8997
Moscow	55,5N	37,3E	0.8908
Kalinigrad	54,7N	20,6E	0.8801
Slough	51,5N	359,4E	0.9167
Kiev	50.5N	35.5E	0.8907
Dourbes	50,1N	4,6E	0.9154
Lannion	48,5N	356,7E	0.9321
Poitiers	46,6N	0,3E	0.9255
Novokazalinsk	45,5N	62,1E	
Rome	41.9N	12.5E	0.8886
Ashkhabad	37,9N	58,3E	0.8397

Table 1. List of the ionosonde stations, their codes, geographic latitudes and longitudes and the correlation coefficient of foF2 with R_{12}.

In the framework of COST actions it is preferred to use only R_{12} as a physical parameter, because its availability and reliability [7]. The inconveniencies arising from the use of a single parameter are eliminated by obtaining different fits to the rising and falling portions of each solar cycle. We obtained the model by first eliminating a parabolic term depending on R12 and then we used the Fourier analysis to model the remaining variation as harmonics of the yearly variation. The following graph gives an idea for the seasonal variation of foF2.

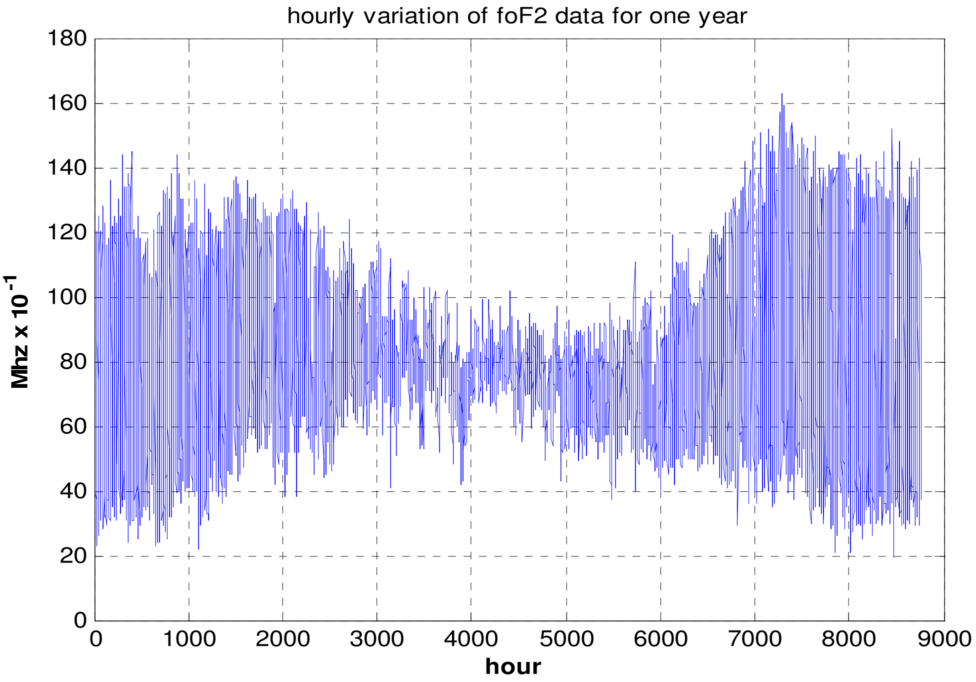


Figure 3. Hourly variation of foF2 data for one year for Dourbes ionosonde station in 1980

In the FFT spectra, the dominant periodicity is a peak at approximately 11 years, which corresponds to the periodicity in sunspot numbers. The subsequent peaks correspond to the yearly variation and its harmonics occurring at 6 month, 4 month and 72 days. These periodicities are due to the non-sinusoidality of the yearly variation, and have no physical meaning, but it is intriguing that the fourth harmonic which has 3 month period is absent while the next harmonic occurring at 72 days is visible.

The coefficients were chosen as linear functions of R_{12} in order to account for the remaining modulation with R_{12}. We have obtained considerable improvement by applying the model to rising and falling portions of the cycle separately. The model for each station involves 3+x2=25 coefficients. The goodness of the model is measured by the l_2 norm of the difference between the data and the model. The average of this discrepancy over all stations is %6.12 if we use 20 years. The repetition of this procedure for rising and falling portions, the resulting errors are about %3 and %4 for rising and falling phases. The lower values of the error for the rising phase agree with [8], who notes that foF2 variations follow R_{12} more closely in the rising phases. The modelling errors are presented together with the prediction errors in Table 2, in the following section.

3. PREDICTION OF MONTHLY MEDIANS OF foF2

The model described in the previous section proved to be more satisfactory when applied to rising and falling phases of solar cycles. Once a mathematical model is obtained, this can be used in principle to predict future values of foF2. Due to the hysteresis effects, predictions based on 3 to 4 year periods give better results. Furthermore, although parabolic and higher order dependencies on R_{12} bring an improvement to the model, linear models behave much well in long term predictions. Based on numerous comparisons of the observation length and order of the model parameters, we arrived to a scheme for the prediction of the monthly median value of foF2 (daily average or for each hour) based on constructing a model on the data of previous 48 months and evaluating the forthcoming month from the model, using the estimated value of R_{12}. This method of using a predetermined period of past data is called a "sliding window" technique. The short term prediction errors are as low as the modelling errors, as shown in the table below.

Station	Falling Cycle 20	Rising Cycle 21	Falling Cycle 21	Rising Cycle 22	1970-1989	Prediction
Ashkhabad	4.13	3.24	4.81	5.90	7.19	3.69
Rome	5.32	7.30	4.68	2.75	7.06	3.45
Poitiers	3.45	2.47	4.43	3.09	4.79	2.52
Lannion	3.73	2.51	4.39	3.31	5.23	2.67
Dourbes	4.01	2.71	4.76	4.19	5.59	3.04
Kiev	4.21	2.90	4.80	3.39	5.87	3.00
Slough	3.89	2.64	4.95	3.25	5.42	2.92
Kaliningrad	3.98	2.87	4.63	3.42	7.36	2.85
Moscow	4.47	3.01	5.09	3.72	5.98	2.80
Sverdlovsk	4.33	2.53	4.62	3.12	5.33	2.91
Uppsala	5.37	3.26	5.44	4.58	7.28	3.47
Arkhangelsk	4.86	3.19	5.07	4.82	6.32	3.17
Average	*4.31*	*3.22*	*4.81*	*3.80*	*6.12*	*3.04*

Table 2. Relative error between measurements and the model for each station, using rising and falling phases of each cycle separately, using 20 year data and using a prediction with a 3-year sliding window.

4.FORECASTING WITH FEEDBACK

The hourly values of foF2 make large amplitude swings in a day. As a typical example we give daily variation curves for Dourbes station.

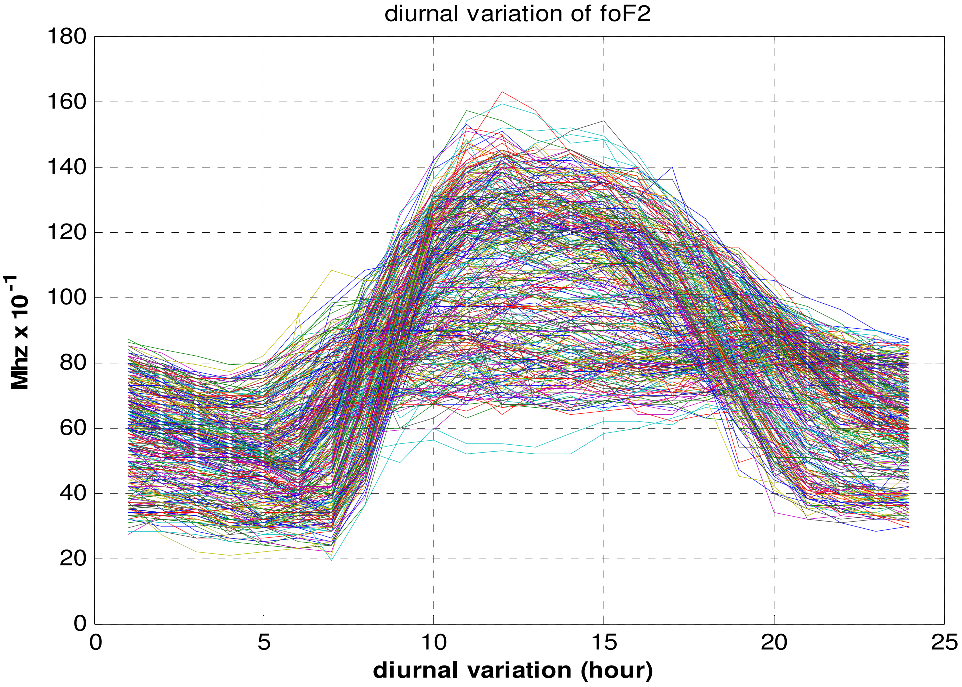


Figure 4. Diurnal variation of foF2 for Dourbes ionosonde station in 1980

In the present work the term “forecasting” is used to designate the estimation of the actual hourly values of foF2 and we use the feedback method in this purpose. In this method, at each step, a measurement error is multiplied by an appropriate constant and it is fed back to modify the controlling signal. In this work the error is the difference between the measured foF2 and the monthly medians. The error of the estimation is multiplied with an appropriate constant and it is added with the reverse sign to correct the forecast at the i+1’st step. That is, if the measurement and median at the time “t_i” are respectively foF2(t_i) and median(t_i) , then the error at stage t_i is

$$E(t_i) = \mathrm{median}(t_i) - \mathrm{foF2}(t_i)$$

As the median value is available at the stage “t_{i+1}” the forecast at the hour “t_{i+1}” is obtained from

$$\mathrm{foF2}(t_{i+1}) = \mathrm{median}(t_{i+1}) - k\,E(t_i)$$

where k is the feedback constant to be determined.

This method has been applied to data for 13 selected stations over 1958-1998 as it is mentioned in the previous section. The comparison of the monthly median data, actual foF2 and forecasted foF2 for one month is shown in Figure.5 .

Forecasting becomes crucial in magnetically disturbed times called “stroms”. In Figure 6 below, we present a detailed view of the forecast for a typical storm time distrubance followed by a quiet day in the same example for Dourbes ionosonde station, for the days 21 and 22 of March 1980.

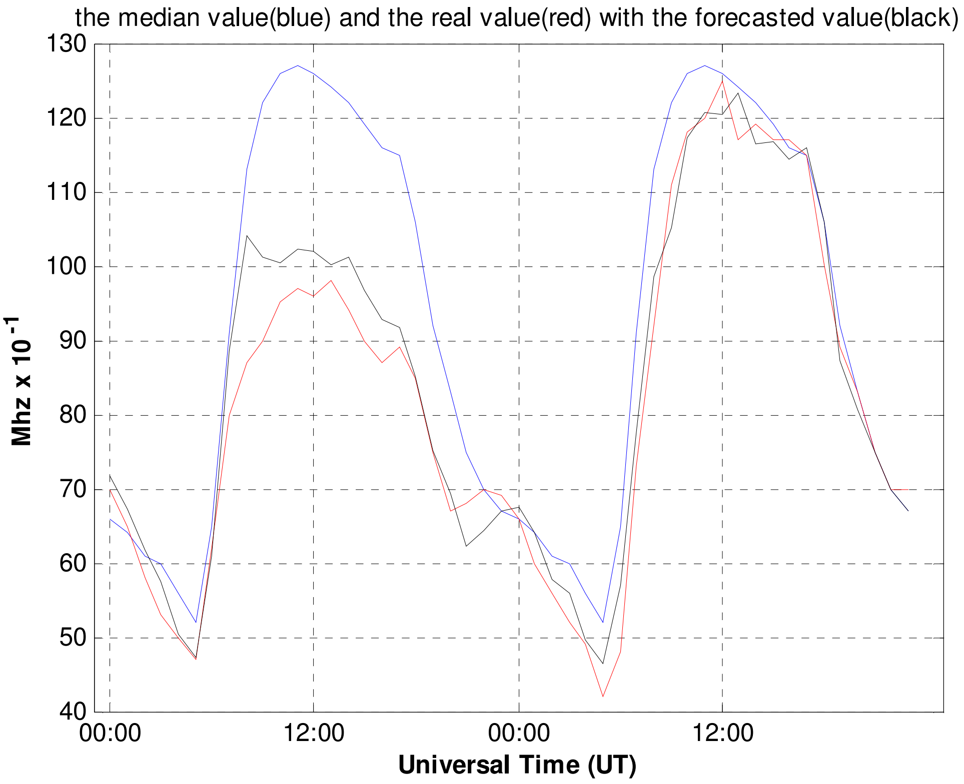


Figure 6 Comparison of monthly medians, hourly data and forecast results. Data shows a typical storm condition in Dourbes 1980 for 2 days from March 21 to March 22.

It can be seen that the forecast by feedback follows the depression at the first day but fails to follow closely the short term fluctuations.

We computed the feedback constant k, by applying feedback with different k values and computing the error with respect to a l_2 norm. The value of k corresponding to local minimum in the chosen norm of the error is called k^*. We allowed k to be in the range $0.0 \le k \le 1.2$ in steps of 0.01 and determined k^* by a one-dimensional optimization algorithm. The optimal feedback coefficients range mostly between 0.7 and 0.9, with a sharp decrease towards larger values. As an example to the selection process, Figure 7 presents the determination of the best feedback constant in the interval 0.0 to 1.2 for Dourbes March 1980.

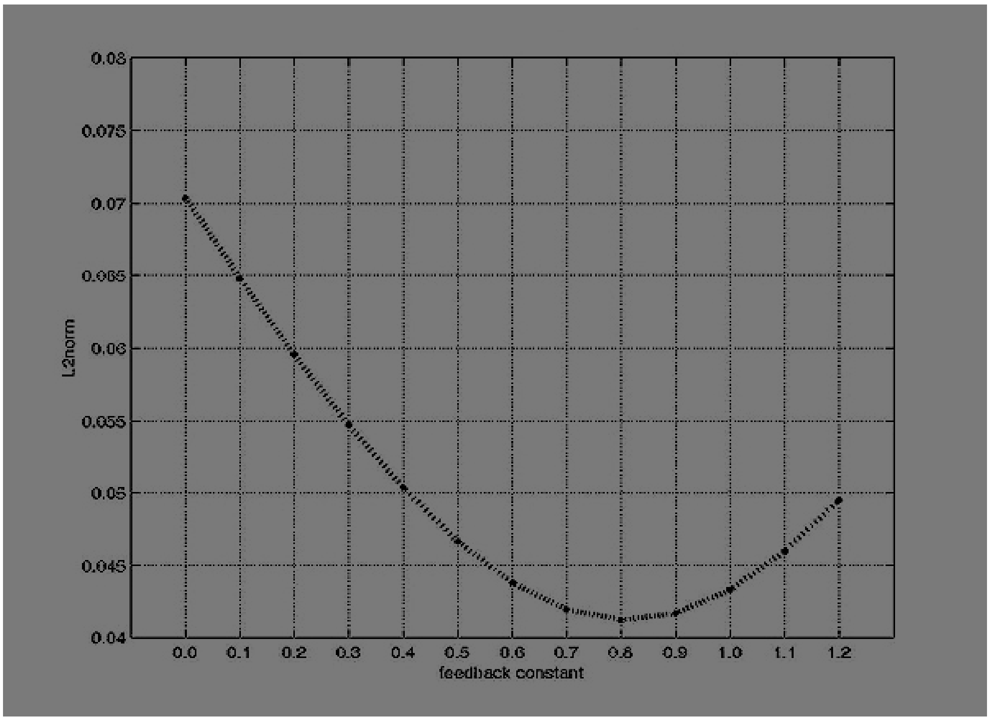
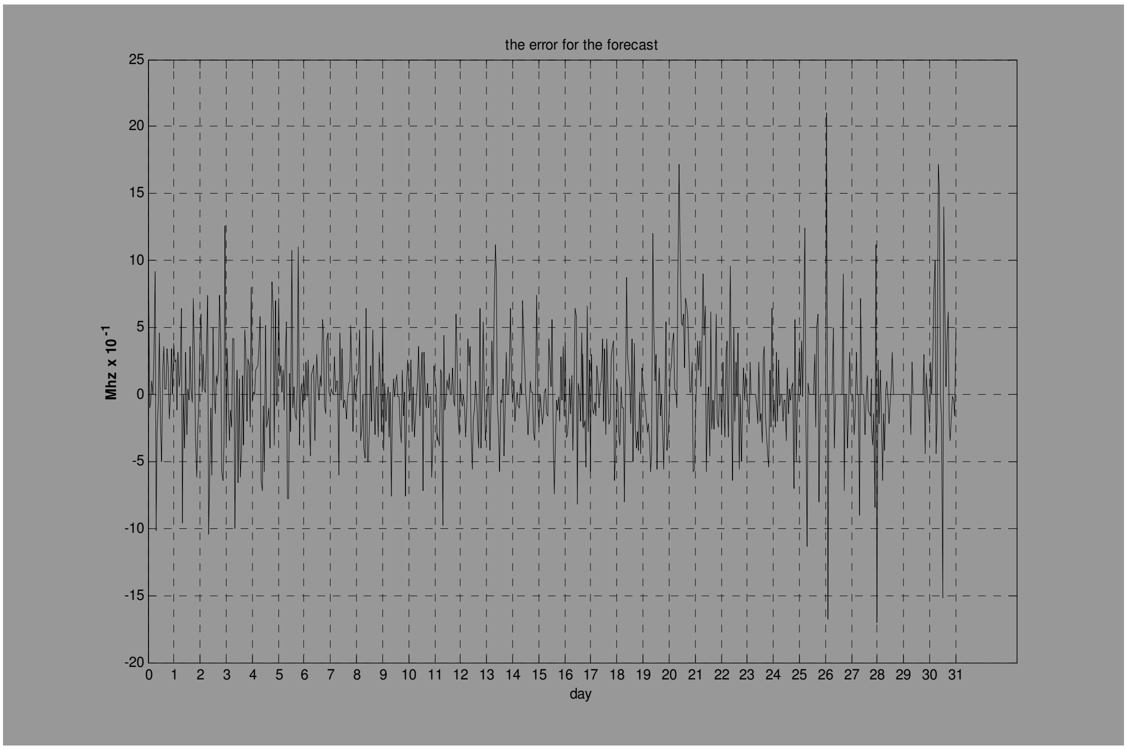


Figure 7. The change of the feedback constant for the selection of k^*. Here for the minimun $l_2$norm= 0.0359, k^* is selected as 0.8.

We have repeated the procedure above for all data samples and presented the results as a table in the Appendix of [6]. A time domain plot of the forecast error with respect to the best feedback constant is shown in Figure 8.

The histogram of errors for ($k = k^*$) and for no feedback ($k = 0.0$) shows that the application of the feedback method reduces considerably to the forecast errors of magnitude > 0.5 Mhz but has a tendency to increase errors below 0.5 Mhz. This situation reflects a typical behaviour presented in Figure 9. The increase in the low amplitude errors have an adverse effect on the total forecast error in the l_2 norm, but these low amplitude deviations are harmless for the determination of reliability bounds, which constitute the ultimate goal of studies on ionospheric parameters.



Figue 8. The variation of the forecast error with respect to time for the best feedback constant in Dourbes ionosone station March 1980.

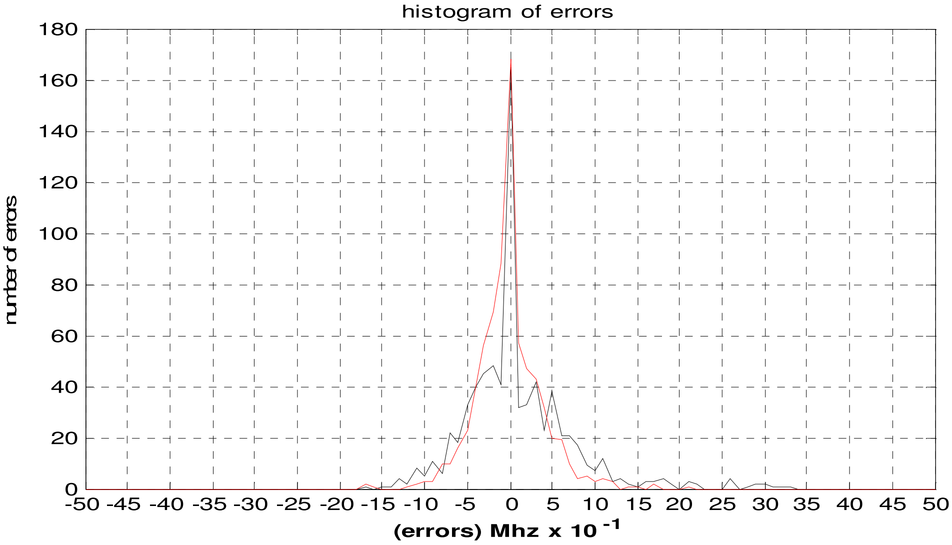


Figure 9. The histogram of the total errors for the best feedback constant(red) and total errors for no feedback constant for Dourbes ionosone station in March 1980

In order to improve the performance, we studied the dependency of the feedback coefficient on months, R_{12} and latitude. We have seen that smaller values of k^* mostly occur in winter time, k^* increases with R_{12} but the latitude dependency do not have a clear rule. These qualitative observations were insufficient to build a mathematical model and we constructed look-up tables for meaningful groupings of the parameter ranges in [6].

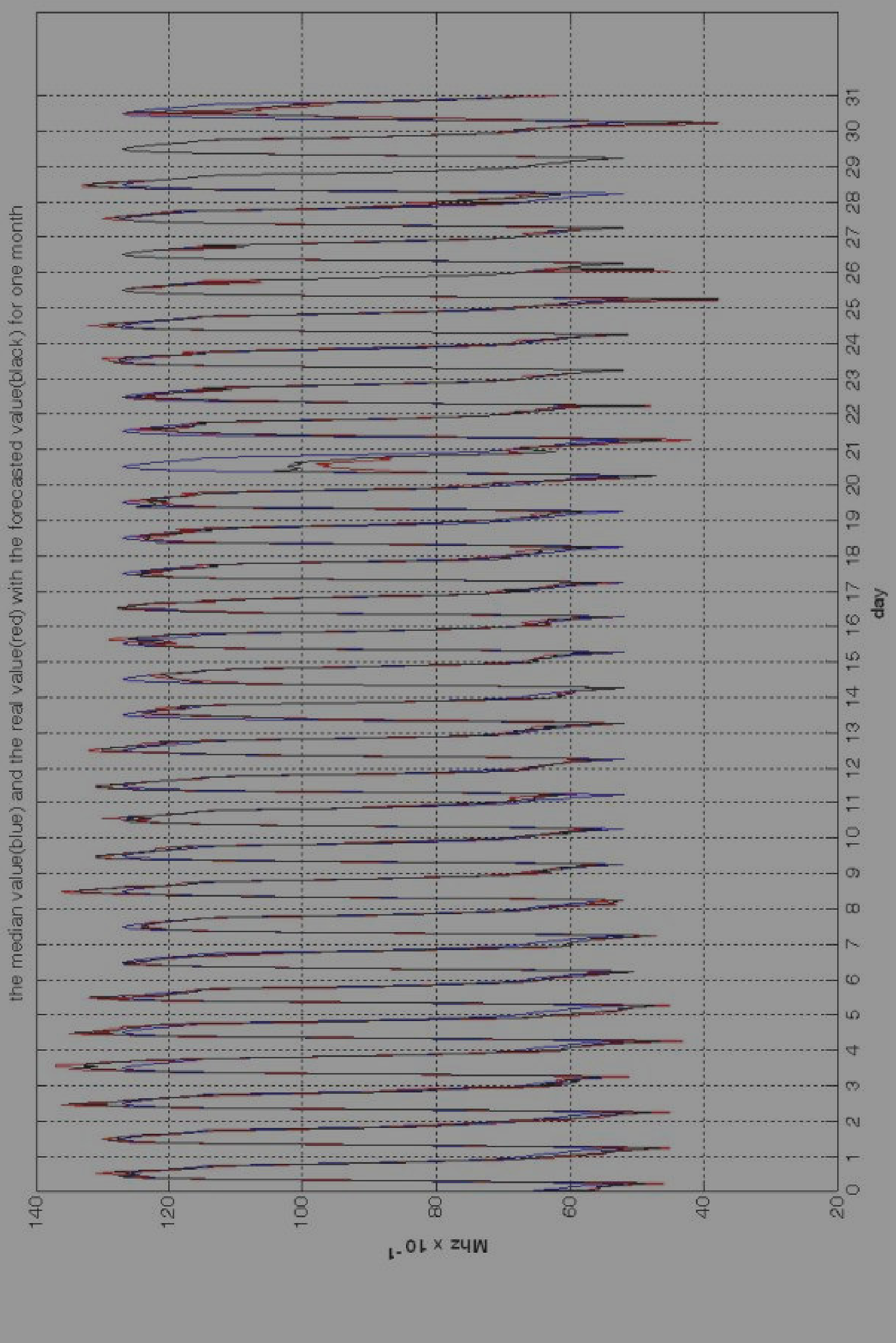


Figure 5 The comparison of the monthly median data (blue), actual foF2 (real values)(red) and forecasted foF2 (black) for Dourbes ionosonde station in March 1980.

In order to present our results in a compact form, we prepared four look-up tables in groups which are separated according to the geographic latitude. In all tables mean *k** values for sunspot maximum and sunspot minimum are given for four seasons. The groups 1-4 involve stations from higer to lower latitudes as shown in the Tables 3-6 below.

Month R_{12}	12 - 01 - 02 Winter	03 - 04 - 05 Spring	06 - 07 - 08 Summer	09 - 10 - 11 Fall
SSN MAXIMUM	0.72	0.8667	0.8667	0.8
SSN MINIMUM	No data	No data	No data	No data

Table 3. Optimal feedback coefficient for stations at latitudes 63^{o}N -70^{o}N

Month R_{12}	12 - 01 - 02 Winter	03 - 04 - 05 Spring	06 - 07 - 08 Summer	09 - 10 - 11 Fall
SSN MAXIMUM	0.75	0.8917	0.8750	0.8583
SSN MINIMUM	0.5	0.8	0.6667	0.7333

Table 4. Optimal feedback coefficient for stations at latitudes 57^{o}N to 63^{o}N

Month R_{12}	12 - 01 - 02 Winter	03 - 04 - 05 Spring	06 - 07 - 08 Summer	09 - 10 - 11 Fall
SSN MAXIMUM	0.7621	0.8943	0.8782	0.8667
SSN MINIMUM	0.6271	0.826	0.7375	0.7823

Table 5. Optimal feedback coefficient for stations at latitudes 51^{o}N – 57^{o}N

Month R_{12}	12 - 01 - 02 Winter	03 - 04 - 05 Spring	06 - 07 - 08 Summer	09 - 10 - 11 Fall
SSN MAXIMUM	0.7064	0.8718	0.8462	0.8269
SSN MINIMUM	0.6090	0.7769	0.691	0.7359

Table 6. Optimal feedback coefficient for stations at latitudes 45^{o}N – 51^{o}N

5.CONCLUSION

In the process of analysing of the ionospheric critical frequency foF2 as a time series, the main practical problem was the conversion between data formats. The handling of multiple files and reading data with unequal number of entries in each line were the biggest challenges. Once the data was put in the MATLAB format, conversion to arrays of various dimensions was flexible and easy and matrix manipularions in least squares approximations and the Fast Fourier Transform (FFT) performed well.

Our experience with handling of this nonstationary data showed that the determination of the appropriate observation interval is crucial in obtaining simple models and powerful estimation tools. The least squares method is then very efficient for obtaining optimal parameters, such as the coefficients of various trigonometric functions, that occur linearly in the model. For the determination of parameters such as fine-tuned periods and optimal feed-back coefficients, it is necessary to use one-dimensional optimization algorithms.

The sliding window tehcnique proved to be very efficient for the prediction of montly medians. Then, the study of the statistical properties of the deviations from monthly medians give reliability bounds discussed in [5]. The feedback method applied to this data first in [2] proved to be useful in forecast and the method was refined in [6].

Those results are aimed to obtain on line mapping of the forecast of foF2 over Europe and it remains to put these results in a user friendly format to produce monthly, daily and hourly forcasts that would run a s stand-alone program.

6. ACKNOWLEDGMENTS

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REFERENCES

[1] S.A. Baykal, A.H. Bilge and Y. Tulunay, “Theoretical spatial and temporal modelling over COST 251 area: Quiet conditions”, (poster), *XXII General Assembly of the European Geophysics Society*, **21-25 April, 1997**.

[2] A.H. Bilge and Y. Tulunay, “A novel on-line method for single station prediction and forecast of the ionospheric critical frequency foF2 1-hour ahead”, *Geophysical Research Letters*, vol **27**, no.9, pp.1383-1386, **2000**.

[3] E.Mizrahi, A.H.Bilge, Y.Tulunay, ”Statistical Properties of the deviations of foF2 from monthly medians”, *Annals of Geophysics*, vol.**45**, n.1, p.131-143, **2002**.

[4] A.H.Bilge, E.Mizrahi, Y.Tulunay, ”Variation of the feedback coefficient with R_{12} and the geographic latitude in 1-h ahead forecast of foF2”, *Annals of Geophysics,* vol.**45**, n.1,p.87-95, **2002**.

[5] A.H.Bilge, E.Mizrahi, Y.Tulunay, ”Reliability bounds for the magnetically quiet time deviations of the ionospheric critical frequency foF2 from monthly medians”, *1st COST 271 Workshop, Sopron-Hungary,* **25-27 September 2001.**

[6] A. Pekcan, “Forecasting of the Ionospherical Critical Frequency for HF Propagation”. M*.Sc. Thesis, Faculty of Aeronautics and Astronautics, Istanbul Technical University*, *2004*.

[7] P.A.. Bradley, “Further study of foF2 and M(3000)F2 in different solar cycles”, *Annali di Geofisica,* vol **37**, pp.201-207, **1994**.

[8] R.P. Kane, “Solar cycle variation of foF2”, *J. Atmos. Terr. Phys.*, vol. **54**, pp. 1201-1205, **1992**.